Resource Allocation for Outdoor-to-Indoor Multicarrier Transmission with Shared UE-side Distributed Antenna Systems

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Abstract—In this paper, we study the resource allocation algorithm design for downlink multicarrier transmission with a shared user equipment (UE)-side distributed antenna system (SUDAS) which utilizes both licensed and unlicensed frequency bands for improving the system throughput. The joint UE selection and transceiver processing matrix design is formulated as a non-convex optimization problem for the maximization of the end-to-end system throughput (bits/s). In order to obtain a tractable resource allocation algorithm, we first show that the optimal transmitter precoding and receiver post-processing matrices jointly diagonalize the end-to-end communication channel. Subsequently, the optimization problem is converted to a scalar optimization problem for multiple parallel channels, which is solved by using an asymptotically optimal iterative algorithm. Simulation results illustrate that the proposed resource allocation algorithm for the SUDAS achieves an excellent system performance and provides a spatial multiplexing gain for single-antenna UEs.

I. INTRODUCTION

Ubiquitous and high data rates are a basic requirement for the next generation wireless communication systems. As a result, orthogonal frequency division multiple access (OFDMA) has been adopted as an air interface for high speed wideband communication systems, due to its flexibility in resource allocation and resistances against multipath fading [1]. On the other hand, multiple-input multiple-output (MIMO) technology has received considerable interest in the past decades as it provides extra degrees of freedom in the spatial domain which facilitates a trade-off between multiplexing gain and diversity gain. Besides, distributed antenna systems (DAS), a special form of MIMO, can be deployed to cover the dead spots in wireless networks, extending service coverage, improving spectral efficiency, and mitigating interference. However, the number of antennas available at the user equipments (UEs) is constrained by the physical size of the devices in practice which leads to a limited spatial multiplexing gain in MIMO systems. As an alternative, multiuser MIMO has been proposed to realize the potential performance gain of MIMO systems by sharing the antennas across the different terminals of a communication system [2], [3]. In [2], the energy efficiency of a three-node multiuser MIMO system was studied for the compress-and-forward protocol. In [3], optimal power allocation was investigated for the maximization of the effective capacity of a multiuser MIMO system

for the case when the receivers collaborate with each other. Recently, there has been a growing interest in combining the concepts of multiuser MIMO, DAS, and OFDMA to improve the performance of wireless communication systems. In [4], the authors studied suboptimal resource allocation algorithm design for multiuser MIMO-OFDMA systems. In [5], a utility-based low complexity scheduling scheme was proposed for multiuser MIMO-OFDMA systems to strike a balance between system throughput and computational complexity. On the other hand, the performance of multiuser MIMO in DAS with limited feedback was investigated in [6]. However, the system performance of the systems [4]–[6] is limited by the system bandwidth which is a very scarce resource in the licensed frequency bands. On the contrary, the unlicensed frequency spectrum around 60 GHz offers a bandwidth of 7 GHz for wireless communications. The utilization of both licensed and unlicensed frequency bands introduces a paradigm shift in system and resource allocation algorithm design due to the resulting new opportunities and challenges. Yet, the potential performance gains of such a hybrid system have not been investigated in the literature.

In this paper, we propose a shared user equipment (UE)-side distributed antenna system (SUDAS) to assist the downlink communication. In particular, SUDAS utilizes both licensed and unlicensed frequency bands simultaneously to facilitate a spatial multiplexing gain for single-antenna receivers. We formulate the resource allocation algorithm design for SUDAS assisted OFDMA downlink transmission systems as a non-convex optimization problem. By exploiting the structure of the optimal base station (BS) precoding and the SUDAS post-processing matrices, the considered matrix optimization problem is transformed into an optimization problem with scalar optimization variables. Capitalizing on this transformation, we develop an iterative algorithm which achieves the asymptotically optimal performance of the proposed SUDAS.

The rest of the paper is organized as follows. In Section II, we outline the model for the considered SUDAS assisted OFDMA downlink transmission system. In Section III, we formulate the resource allocation as a non-convex optimization problem. Simulation results for the performance of the proposed algorithm are presented in Section IV. In Section V, we conclude with a brief summary of our results.

II. SUDAS ASSISTED OFDMA NETWORK MODEL

In this section, after introducing the notation used in this paper, we present the adopted channel and signal models.

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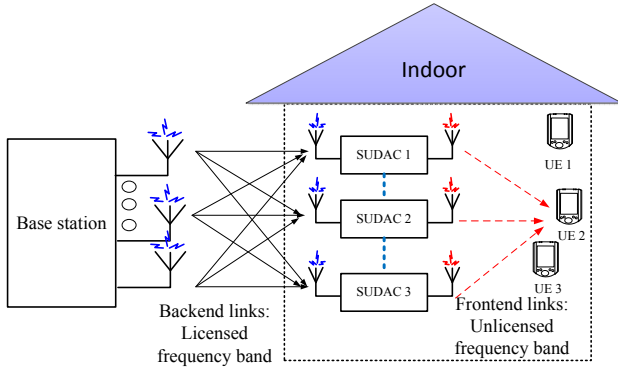
Fig. 1. Downlink communication system model with a base station (BS), $K = 3$ user equipments (UEs), and $M = 3$ SUDACs. The backend links use a licensed frequency band and the frontend links use an unlicensed frequency band such as the millimeter wave band (e.g. ~ 60 GHz).

Fig. 2. Illustration of signal forwarding from the licensed band to different unlicensed frequency sub-bands in the SUDAS.

A. Notation

We use boldface capital and lower case letters to denote matrices and vectors, respectively. $\mathbf{A}^H$, $\det(\mathbf{A})$, $\mathrm{Tr}(\mathbf{A})$, and $\mathrm{Rank}(\mathbf{A})$ represent the Hermitian transpose, determinant, trace, and rank of matrix $\mathbf{A}$; $\mathbf{A} \succeq \mathbf{0}$ indicates that $\mathbf{A}$ is a positive semidefinite matrix; $\mathbf{I}_N$ is the $N \times N$ identity matrix; $\mathbb{C}^{N \times M}$ denotes the set of all $N \times M$ matrices with complex entries; $\mathbb{H}^N$ denotes the set of all $N \times N$ Hermitian matrices; $\mathrm{diag}(x_1, \cdots, x_K)$ denotes a diagonal matrix with the diagonal elements given by $\{x_1, \cdots, x_K\}$; the circularly symmetric complex Gaussian (CSCG) distribution with mean μ and variance σ^2 is denoted by $\mathcal{CN}(\mu, \sigma^2)$; $\sim$ stands for "distributed as"; $[x]^+$ returns 0 when $x < 0$ and returns x if $x \geq 0$; $\mathcal{E}\{\cdot\}$ denotes statistical expectation.

B. SUDAS Downlink Communication Model

We consider a SUDAS assisted OFDMA downlink transmission network which consists of one BS, a SUDAS, and K UEs, cf. Figure 1. A SUDAS consists of M shared user equipment (UE)-side distributed antenna components (SUDACs). A SUDAC is a small and cheap device[1] which utilizes both a licensed and an unlicensed frequency band simultaneously for increasing the end-to-end communication data rate. Conceptually, a basic SUDAC is equipped with one antenna for use in a licensed band and one antenna for use in an unlicensed band. Besides, a SUDAC is equipped with a mixer to perform frequency up-conversion/down-conversion. Specifically, for the downlink, the SUDAC first receives the signal from the BS in a licensed frequency band (backend link). Then the SUDAC processes the received signal and forwards the signal to the UEs in an unlicensed frequency band (frontend link). We note that since signal reception and transmission at each SUDAC are separated in frequency, cf. Figure 2, simultaneous signal reception and transmission can be performed which is not possible for traditional relaying systems[2] due to the limited spectrum availability in the li-

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[1]In practice, a SUDAC could be integrated into electrical devices such as electrical wall outlets, switches, and light outlets.

[2]Since the BS-to-SUDAS and SUDAS-to-UE links operate on two different frequencies, the SUDAS should not be considered a relaying system [7].
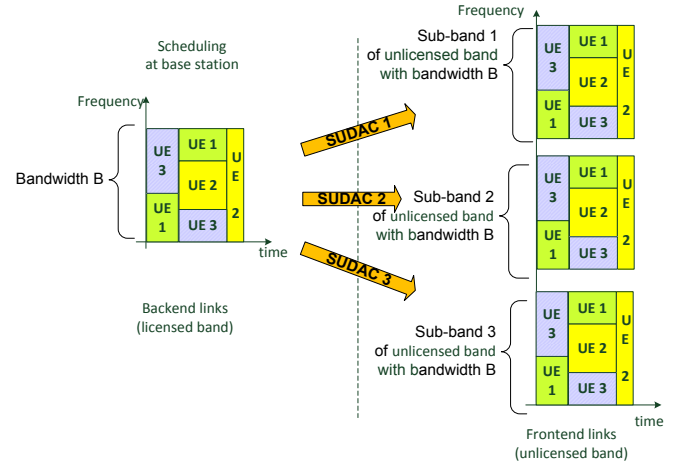
censed bands. In practice, a huge bandwidth is available in the unlicensed bands. For instance, there is nearly 7 GHz of unlicensed frequency spectrum available for information transmission in the $57 - 64$ GHz band (millimeter wave band). In this paper, we study the potential system performance gain for outdoor-to-indoor transmission achieved by the SUDAS. We assume that the SUDACs are installed in electrical wall outlets and can cooperate with each other by sharing the channel state information and received signals, i.e., $\mathbf{y}_{\mathrm{S}}^{[i,k]}$, via low data-rate power line communication links. In other words, joint processing between SUDACs is possible such that the SUDACs can fully utilize their antennas[3]. Besides, the UEs only listen to the unlicensed frequency band.

C. SUDAS Downlink Communication Channel Model

The BS is equipped with N_T transmit antennas transmitting signals in a licensed frequency band. The UEs are single-antenna devices receiving the signal in the unlicensed frequency band. We focus on a wideband multicarrier communication system with n_F subcarriers. The communication channel is time-invariant within a scheduling slot. The BS performs spatial multiplexing in the licensed band. The data symbol vector $\mathbf{s}^{[i,k]} \in \mathbb{C}^{N_\mathrm{S} \times 1}$ on subcarrier $i \in \{1, \ldots, n_F\}$ for UE k is precoded at the BS as

$$\mathbf{x}^{[i,k]} = \mathbf{P}^{[i,k]} \mathbf{s}^{[i,k]}, \tag{1}$$

where $\mathbf{P}^{[i,k]} \in \mathbb{C}^{N_\mathrm{T} \times N_\mathrm{S}}$ is the precoding matrix adopted by the BS on subcarrier i. The signals received on subcarrier i at the M SUDACs for UE k are given by

$$\mathbf{y}_{\mathrm{S}}^{[i,k]} = \mathbf{H}_{\mathrm{B} \to \mathrm{S}}^{[i]} \mathbf{x}^{[i,k]} + \mathbf{z}_m^{[i]}, \tag{2}$$

where $\mathbf{y}_{\mathrm{S}}^{[i,k]} = [y_{\mathrm{S}_1}^{[i,k]} \ldots y_{\mathrm{S}_m}^{[i,k]} \ldots y_{\mathrm{S}_M}^{[i,k]}]^T$ and $y_{\mathrm{S}_m}^{[i,k]} \in \{1, \ldots, M\}$ denotes the received signal at SUDAC m. $\mathbf{H}_{\mathrm{B} \to S}^{[i]}$ is the $M \times N_\mathrm{T}$ MIMO channel matrix between the BS and the

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[3]We note that different SUDAS configurations are possible in practice including non-cooperative SUDACs. In this paper, we are interested in the case when the SUDACs are willing to cooperate to explore the maximum system performance.

M SUDACs on subcarrier i and captures the joint effects of path loss, shadowing, and multi-path fading. $\mathbf{z}^{[i,k]}$ is the additive white Gaussian noise (AWGN) vector with distribution $\mathcal{CN}(\mathbf{0}, \mathbf{\Sigma})$ on subcarrier i impairing the M SUDACs where $\mathbf{\Sigma}$ is an $M \times M$ diagonal covariance matrix with each main diagonal element given by N_0.

Then, each SUDAC performs frequency repetition in the unlicensed band. In particular, the M SUDACs multiply the received signal vector on subcarrier i by $\mathbf{F}^{[i,k]} \in \mathbb{C}^{M \times M}$ and forward the processed signal vector to UE k on subcarrier i in M different independent frequency sub-bands in the unlicensed spectrum, cf. Figure 2. In other words, each SUDAC forwards its received signal in a different sub-band and thereby avoids further multiple access interference in the unlicensed spectrum.

Then, the signal received at UE k on subcarrier i from the SUDACs in the M frequency bands, $\mathbf{y}_{S \to UE}^{[i,k]} \in \mathbb{C}^{M \times 1}$, can be expressed as

$$\mathbf{y}_{S \to UE}^{[i,k]} \tag{3}$$
$$= \mathbf{H}_{S \to UE}^{[i,k]} \mathbf{F}^{[i,k]} \Big(\mathbf{H}_{B \to S}^{[i]} \mathbf{x}^{[i,k]} + \mathbf{z}^{[i]} \Big) + \mathbf{n}^{[i,k]}$$
$$= \underbrace{\mathbf{H}_{S \to UE}^{[i,k]} \mathbf{F}^{[i,k]} \mathbf{H}_{B \to S}^{[i]} \mathbf{P}^{[i,k]} \mathbf{s}^{[i,k]}}_{\text{desired signal}} + \underbrace{\mathbf{H}_{S \to UE}^{[i,k]} \mathbf{F}^{[i,k]} \mathbf{z}^{[i]}}_{\text{amplified noise}} + \mathbf{n}^{[i,k]}.$$

The m-th element of vector $\mathbf{y}_{S \to UE}^{[i,k]}$ represents the received signal at UE k in the m-th unlicensed frequency sub-band. Besides, since the SUDACs forward the received signals in different frequency bands, $\mathbf{H}_{S \to UE}^{[i,k]}$ is a diagonal matrix with the diagonal elements representing the channel gain between the SUDACs and UE k on subcarrier i in the unlicensed sub-band m. $\mathbf{n}^{[i,k]} \in \mathbb{C}^{M \times 1}$ is the AWGN vector at UE k on subcarrier i with distribution $\mathcal{CN}(\mathbf{0}, \mathbf{\Sigma}_k)$. $\mathbf{\Sigma}_k$ is an $M \times M$ diagonal matrix and each main diagonal element is equal to N_0. In order to simplify the subsequent mathematical expressions and without loss of generality, we adopt in the following a normalized noise variance of $N_0 = 1$ at all receive antennas of the SUDACs and the UEs.

We assume that $M \geq N_S$ and UE k employs a linear receiver for estimating the data vector symbol received in the M different frequency bands in the unlicensed band. The estimated data vector symbols, $\hat{\mathbf{s}}^{[i,k]} \in \mathbb{C}^{N_S \times 1}$, on subcarrier i is given by:

$$\hat{\mathbf{s}}^{[i,k]} = (\mathbf{W}^{[i,k]})^H \mathbf{y}_{S \to UE}^{[i,k]}, \tag{4}$$

where $\mathbf{W}^{[i,k]} \in \mathbb{C}^{M \times M}$ is a post-processing matrix used for subcarrier i at UE k. Without loss of generality, we assume that $\mathcal{E}\{\mathbf{s}^{[i,k]}(\mathbf{s}^{[i,k]})^H\} = \mathbf{I}_{N_S}$. As a result, the mean square error (MSE) matrix for the data transmission on subcarrier i for UE k via the SUDAS and the optimal post processing matric are given by

$$\mathbf{E}^{[i,k]} = \mathcal{E}\{(\hat{\mathbf{s}}^{[i,k]} - \mathbf{s}^{[i,k]})(\hat{\mathbf{s}}^{[i,k]} - \mathbf{s}^{[i,k]})^H\}$$
$$= \Big[\mathbf{I}_{N_S} + (\mathbf{\Gamma}^{[i,k]})^H (\mathbf{\Theta}^{[i,k]})^{-1} (\mathbf{\Gamma}^{[i,k]}) \Big]^{-1}, \tag{5}$$
$$\text{and } \mathbf{W}^{[i,k]} = (\mathbf{\Gamma}^{[i,k]} (\mathbf{\Gamma}^{[i,k]})^H + \mathbf{\Theta}^{[i,k]})^{-1} \mathbf{\Gamma}^{[i,k]}, \tag{6}$$

respectively, where $\mathbf{\Gamma}^{[i,k]}$ is the effective MIMO channel matrix between the BS and UE k via the SUDAS on subcarrier

i, and $\mathbf{\Theta}^{[i,k]}$ is the corresponding equivalent noise covariance matrix. These matrices are given by

$$\mathbf{\Gamma}^{[i,k]} = \mathbf{H}_{S \to UE}^{[i,k]} \mathbf{F}^{[i,k]} \mathbf{H}_{B \to S}^{[i]} \mathbf{P}^{[i,k]} \quad \text{and}$$
$$\mathbf{\Theta}^{[i,k]} = \Big(\mathbf{H}_{S \to UE}^{[i,k]} \mathbf{F}^{[i,k]} \Big) \Big(\mathbf{H}_{S \to UE}^{[i,k]} \mathbf{F}^{[i,k]} \Big)^H + \mathbf{I}_M. \tag{7}$$

III. RESOURCE ALLOCATION AND SCHEDULING DESIGN

In this section, we first introduce the adopted system performance measure. Then, the resource allocation and scheduling design is formulated as an optimization problem.

A. System Throughput

The end-to-end achievable data rate $R^{[i,k]}$ on subcarrier i between the BS and UE k via the SUDAS is given by [8]

$$R^{[i,k]} = -\log_2 \Big(\det[\mathbf{E}^{[i,k]}] \Big). \tag{8}$$

The data rate (bit/s) delivered to UE k can be expressed as

$$\rho^{[k]} = \sum_{i=1}^{n_F} s^{[i,k]} R^{[i,k]}, \tag{9}$$

where $s^{[i,k]} \in \{0, 1\}$ is the binary subcarrier allocation indicator. The weighted system throughput via the SUDAS is given by

$$\text{TP}(\mathcal{P}, \mathcal{S}) = \sum_{k=1}^{K} w^{[k]} \rho^{[k]}, \tag{10}$$

where $\mathcal{P} = \{\mathbf{P}^{[i,k]}, \mathbf{F}^{[i,k]}\}$ and $\mathcal{S} = \{s^{[i,k]}\}$ are the precoding and subcarrier allocation policies, respectively. $w^{[k]}$ is a positive constant which indicates the priority of different UEs. It is known that by adjusting the values of $w^{[k]}$, different kinds of fairness such as max-min fairness and proportional fairness can be achieved [9], [10].

B. Problem Formulation

The optimal precoding matrices, $\mathcal{P}^* = \{\mathbf{P}^{[i,k]*}, \mathbf{F}^{[i,k]*}\}$, and the optimal subcarrier allocation policy, $\mathcal{S}^* = \{s^{[i,k]*}\}$, can be obtained by solving the following optimization problem:

$$\underset{\mathcal{P}, \mathcal{S}}{\text{maximize}} \quad \text{TP}(\mathcal{P}, \mathcal{S})$$

$$\text{s.t.} \quad \text{C1:} \quad \sum_{k=1}^{K} \sum_{i=1}^{n_F} s^{[i,k]} \text{Tr} \Big(\mathbf{P}^{[i,k]} (\mathbf{P}^{[i,k]})^H \Big) \leq P_T,$$
$$\text{C2:} \quad \sum_{k=1}^{K} \sum_{i=1}^{n_F} s^{[i,k]} \text{Tr} \Big(\mathbf{G}^{[i,k]} \Big) \leq M P_{\max},$$
$$\text{C3:} \quad \sum_{k=1}^{K} s^{[i,k]} \leq 1, \quad \forall i,$$
$$\text{C4:} \quad s^{[i,k]} \in \{0, 1\}, \quad \forall i, k, \tag{11}$$

where $\text{Tr} \Big(\mathbf{G}^{[i,k]} \Big)$ is the total power transmitted by the SUDAS on subcarrier i for UE k and

$$\mathbf{G}^{[i,k]} = \mathbf{F}^{[i,k]} \Big(\mathbf{H}_{B \to S}^{[i]} \mathbf{P}^{[i,k]} (\mathbf{P}^{[i,k]})^H (\mathbf{H}_{B \to S}^{[i]})^H + \mathbf{I}_M \Big) (\mathbf{F}^{[i,k]})^H.$$

Constants P_T and MP_{max} in C1 and C2 are the maximum transmit power allowances for the BS and the SUDAS (M SUDACs), respectively, where P_{max} is the average transmit power budget for a SUDAC. Constraints C3 and C4 are imposed to guarantee that each subcarrier serves at most one UE.

C. Transformation of the Optimization Problem

The considered optimization problem consists of a non-convex objective function and combinatorial constraints which do not facilitate a tractable resource allocation algorithm design. In order to obtain an efficient resource allocation algorithm, we study the structure of the optimal precoding policy. For this purpose, we now define the following matrices before stating an important theorem concerning the structure of the optimal precoding matrices. Using singular value decomposition (SVD), the channel matrices $\mathbf{H}_{B \to S}^{[i]}$ and $\mathbf{H}_{S \to UE}^{[i,k]}$ can be written as

$$\begin{aligned}
\mathbf{H}_{B \to S}^{[i]} &= \mathbf{U}_{B \to S}^{[i]} \boldsymbol{\Lambda}_{B \to S}^{[i]} (\mathbf{V}_{B \to S}^{[i]})^H \text{ and} \\
\mathbf{H}_{S \to UE}^{[i,k]} &= \mathbf{U}_{S \to UE}^{[i,k]} \boldsymbol{\Lambda}_{S \to UE}^{[i,k]} (\mathbf{V}_{S \to UE}^{[i,k]})^H,
\end{aligned} \quad (12)$$

respectively, where $\mathbf{U}_{B \to S}^{[i]} \in \mathbb{C}^{M \times M}, \mathbf{V}_{B \to S}^{[i]} \in \mathbb{C}^{N_T \times N_T}, \mathbf{U}_{S \to UE}^{[i,k]} \in \mathbb{C}^{M \times M}$, and $\mathbf{V}_{S \to UE}^{[i,k]} \in \mathbb{C}^{M \times M}$ are unitary matrices. $\boldsymbol{\Lambda}_{B \to S}^{[i]}$ and $\boldsymbol{\Lambda}_{S \to UE}^{[i,k]}$ and are $M \times N_T$ and $M \times M$ matrices with main diagonal element vectors $\mathrm{diag}(\boldsymbol{\Lambda}_{B \to S}^{[i]}) = \left[\sqrt{\gamma_{B \to S,1}^{[i]}} \sqrt{\gamma_{B \to S,2}^{[i]}} \cdots \sqrt{\gamma_{B \to S,R_1}^{[i]}} \right]$ and $\mathrm{diag}(\boldsymbol{\Lambda}_{S \to UE}^{[i,k]}) = \left[\sqrt{\gamma_{S \to UE,1}^{[i,k]}} \sqrt{\gamma_{S \to UE,2}^{[i,k]}} \cdots \sqrt{\gamma_{S \to UE,R_2}^{[i,k]}} \right]$, where the elements are ordered in ascending order, respectively. $R_1 = \mathrm{Rank}(\mathbf{H}_{B \to S}^{[i]})$ and $R_2 = \mathrm{Rank}(\mathbf{H}_{S \to UE}^{[i,k]})$. Variables $\gamma_{B \to S,n}^{[i]}$ and $\gamma_{S \to UE,n}^{[i,k]}$ represent the equivalent channel-to-noise ratio (CNR) on spatial channel n in subcarrier i of the BS-to-SUDAS channel and the SUDAS-to-UE k channel, respectively.

We are now ready to introduce the following theorem.

Theorem 1: Suppose that $\mathrm{Rank}(\mathbf{P}^{[i,k]}) = \mathrm{Rank}(\mathbf{F}^{[i,k]}) = N_S \leq \min\{\mathrm{Rank}(\mathbf{H}_{S \to UE}^{[i]}), \mathrm{Rank}(\mathbf{H}_{B \to S}^{[i]})\}$. In this case, the optimal linear precoding matrices used at the BS and the SU-DACs jointly diagonalize the BS-to-SUDAS-to-UE channels on each subcarrier, despite the non-convexity of the objective function. The optimal precoding matrices are given by

$$\begin{aligned}
\mathbf{P}^{[i,k]} &= \widetilde{\mathbf{V}}_{B \to S}^{[i]} \boldsymbol{\Lambda}_B^{[i,k]} \text{ and} \\
\mathbf{F}^{[i,k]} &= \widetilde{\mathbf{V}}_{S \to UE}^{[i,k]} \boldsymbol{\Lambda}_F^{[i,k]} (\widetilde{\mathbf{U}}_{B \to S}^{[i,k]})^H
\end{aligned} \quad (13)$$

respectively, where $\widetilde{\mathbf{V}}_{B \to S}^{[i]}$, $\widetilde{\mathbf{V}}_{S \to UE}^{[i,k]}$, and $\widetilde{\mathbf{U}}_{B \to S}^{[i,k]}$ are the N_S rightmost columns of $\mathbf{V}_{B \to S}^{[i]}$, $\mathbf{V}_{S \to UE}^{[i,k]}$, and $\mathbf{U}_{B \to S}^{[i,k]}$, respectively. Matrices $\boldsymbol{\Lambda}_B^{[i,k]} \in \mathbb{C}^{N_S \times N_S}$ and $\boldsymbol{\Lambda}_F^{[i,k]} \in \mathbb{C}^{N_S \times N_S}$ are diagonal matrices with diagonal element vectors $\mathrm{diag}(\boldsymbol{\Lambda}_B^{[i,k]}) = \left[\sqrt{P_{B \to S,1}^{[i,k]}} \cdots \sqrt{P_{B \to S,n}^{[i,k]}} \cdots \sqrt{P_{B \to S,N_S}^{[i,k]}} \right]$, and $\mathrm{diag}(\boldsymbol{\Lambda}_F^{[i,k]}) = \left[\sqrt{P_{S \to UE,1}^{[i,k]}} \cdots \sqrt{P_{S \to UE,n}^{[i,k]}} \cdots \sqrt{P_{S \to UE,N_S}^{[i,k]}} \right]$, respectively. Variables $P_{B \to S,n}^{[i,k]}$ and $P_{S \to UE,n}^{[i,k]}$ are, respectively, the equivalent transmit powers of the BS-to-SUDAS link and

the SUDAS-to-UE link for UE k on spatial channel n in subcarrier i.

Proof: Please refer to the Appendix. ∎

By Theorem 1, the end-to-end MIMO channel on subcarrier i is converted into N_S parallel spatial channels if the optimal precoding matrices are used.

Therefore, the achievable rate on subcarrier i between the BS and UE k via the SUDAS can be expressed as [11], [12]:

$$R^{[i,k]} = \sum_{n=1}^{N_S} \log_2(1 + \mathrm{SINR}_n^{[i,k]}) \quad \text{where} \quad (14)$$

$$\mathrm{SINR}_n^{[i,k]} = \frac{\gamma_{B \to S,n}^{[i]} P_{B \to S,n}^{[i,k]} P_{S \to UE,n}^{[i,k]} \gamma_{S \to UE,n}^{[i,k]}}{1 + \gamma_{B \to S,n}^{[i]} P_{B \to S,n}^{[i,k]} + P_{S \to UE,n}^{[i,k]} \gamma_{S \to UE,n}^{[i,k]}}$$

is the received signal-to-interference-plus-noise-ratio (SINR). Although the objective function is now a scalar function with respect to the optimization variables, it is still non-convex. To obtain a tractable resource allocation algorithm design, we propose the following objective function approximation. In particular, the end-to-end SINR on subcarrier i for UE k can be approximated as

$$\mathrm{SINR}_n^{[i,k]} \approx \widetilde{\mathrm{SINR}}_n^{[i,k]} \quad \text{where} \quad (15)$$

$$\widetilde{\mathrm{SINR}}_n^{[i,k]} = \frac{\gamma_{B \to S,n}^{[i]} P_{B \to S,n}^{[i,k]} P_{S \to UE,n}^{[i,k]} \gamma_{S \to UE,n}^{[i,k]}}{\gamma_{B \to S,n}^{[i]} P_{B \to S,n}^{[i,k]} + P_{S \to UE,n}^{[i,k]} \gamma_{S \to UE,n}^{[i,k]}}. (16)$$

We note that this approximation is asymptotically tight in high SNR. The next step is to handle the combinatorial constraint C4 in (11). To this end, we adopt the time-sharing relaxation approach. In particular, we relax $s^{[i,k]}$ in C4 such that it is a real valued optimization variable between zero and one [13], [14], i.e., $0 \leq s^{[i,k]} \leq 1$. It is shown in [14] that the time-sharing relaxation is asymptotically optimal for a sufficient number of subcarriers[4]. Next, we define two auxiliary optimization variables $\tilde{P}_{B \to S,n}^{[i,k]} = s^{[i,k]} P_{B \to S,n}^{[i,k]}$ and $\tilde{P}_{S \to UE,n}^{[i,k]} = s^{[i,k]} P_{S \to UE,n}^{[i,k]}$ and rewrite the optimization problem as:

$$\underset{\widetilde{P}_{B \to S}^{[i,k]}, \widetilde{P}_{S \to UE,n}^{[i,k]}, \mathcal{S}}{\text{maximize}} \sum_{k=1}^{K} \sum_{i=1}^{N_F} \sum_{n=1}^{N_S} s^{[i,k]} \log_2 \left(1 + \frac{\overline{\mathrm{SINR}}_n^{[i,k]}}{s^{[i,k]}} \right)$$

s.t. C1: $\sum_{k=1}^{K} \sum_{i=1}^{n_F} \sum_{n=1}^{N_S} \widetilde{P}_{B \to S,n}^{[i,k]} \leq P_T$,

C2: $\sum_{k=1}^{K} \sum_{i=1}^{n_F} \sum_{n=1}^{N_S} \widetilde{P}_{S \to UE,n}^{[i,k]} \leq MP_{max}$,

C3: $\sum_{k=1}^{K} s^{[i,k]} \leq 1, \quad \forall i, \quad$ C4: $0 \leq s^{[i,k]} \leq 1, \quad \forall i, k,$

C5: $\widetilde{P}_{B \to S,n}^{[i,k]}, \widetilde{P}_{S \to UE,n}^{[i,k]} \geq 0, \quad \forall i, k, n, \quad (16)$

where $\overline{\mathrm{SINR}}_n^{[i,k]} = \widetilde{\mathrm{SINR}}_n^{[i,k]} \Big|_{\Phi}$ and $\Phi = \{P_{B \to S,n}^{[i,k]} = \tilde{P}_{B \to S,n}^{[i,k]}/s^{[i,k]}, P_{S \to UE,n}^{[i,k]} = \tilde{P}_{S \to UE,n}^{[i,k]}/s^{[i,k]}\}$. It can be shown

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[4]It has been shown by simulation in [15] that the performance gap due to time-sharing relaxation is virtually zero even for OFDMA systems with only 8 subcarriers.

$$P_{\text{B}\to\text{S},n}^{[i,k]} = \left[\frac{\gamma_{\text{S}\to\text{UE},n}^{[i,k]} P_{\text{S}\to\text{UE},n}^{[i,k]} \left(\frac{\sqrt{4w^{[k]}\gamma_{\text{B}\to\text{S},n}^{[i]}(1+\gamma_{\text{S}\to\text{UE},n}^{[i,k]} P_{\text{S}\to\text{UE},n}^{[i,k]})+(\gamma_{\text{S}\to\text{UE},n}^{[i,k]})^2\lambda(P_{\text{S}\to\text{UE},n}^{[i,k]})^2\ln(2)}}{\sqrt{\lambda}\sqrt{\ln(2)}} - \gamma_{\text{S}\to\text{UE},n}^{[i,k]} P_{\text{S}\to\text{UE},n}^{[i,k]} - 2 \right)}{2(\gamma_{\text{B}\to\text{S},n}^{[i]}\gamma_{\text{S}\to\text{UE},n}^{[i,k]} P_{\text{S}\to\text{UE},n}^{[i,k]} + \gamma_{\text{B}\to\text{S},n}^{[i]})} \right]^+ \quad (17)$$

$$P_{\text{S}\to\text{UE},n}^{[i,k]} = \left[\frac{\gamma_{\text{B}\to\text{S},n}^{[i]} P_{\text{B}\to\text{S},n}^{[i,k]} \left(\frac{\sqrt{(\gamma_{\text{B}\to\text{S},n}^{[i]})^2\beta(P_{\text{B}\to\text{S},n}^{[i,k]})^2\ln(2)+(\gamma_{\text{B}\to\text{S},n}^{[i]} P_{\text{B}\to\text{S},n}^{[i,k]}+1)4w^{[k]}\gamma_{\text{S}\to\text{UE},n}^{[i,k]}}}{\sqrt{\beta}\sqrt{\ln(2)}} - \gamma_{\text{B}\to\text{S},n}^{[i]} P_{\text{B}\to\text{S},n}^{[i,k]} - 2 \right)}{2(\gamma_{\text{B}\to\text{S},n}^{[i]}\gamma_{\text{S}\to\text{UE},n}^{[i,k]} P_{\text{B}\to\text{S},n}^{[i,k]} + \gamma_{\text{S}\to\text{UE},n}^{[i,k]})} \right]^+ \quad (18)$$

TABLE I
ITERATIVE RESOURCE ALLOCATION ALGORITHM

Algorithm Alternating Optimization

1: Initialize the maximum number of iterations $L_{\max}$ and a small constant $\kappa \to 0$
2: Set iteration index $l = 0$ and initialize a feasible solution point $P_{\text{B}\to\text{S},n}^{[i,k]}(l), P_{\text{S}\to\text{UE},n}^{[i,k]}(l), s^{[i,k]}(l), l = l+1$
3: **repeat** {Loop}
4:    Solve (16) for $P_{\text{B}\to\text{S},n}^{[i,k]}$ and $s^{[i,k]}$ with a fixed $P_{\text{S}\to\text{UE},n}^{[i,k]}(l-1)$ by using (17) and (19) which leads to intermediate power allocation variables $P_{\text{B}\to\text{S},n}^{[i,k]'}$ and a subcarrier allocation policy $s^{[i,k]'}$
5:    Solve (16) for $P_{\text{S}\to\text{UE},n}^{[i,k]}$ and $s^{[i,k]}$ with $P_{\text{B}\to\text{S},n}^{[i,k]'}$ via equation (18) and (19) which leads to intermediate power allocation variables $P_{\text{S}\to\text{UE},n}^{[i,k]'}$ and a subcarrier allocation policy $s^{[i,k]''}$
6:    **if** $|P_{\text{S}\to\text{UE},n}^{[i,k]'} - P_{\text{S}\to\text{UE},n}^{[i,k]}(l-1)| \leq \kappa$ and $|P_{\text{S}\to\text{UE},n}^{[i,k]'} - P_{\text{S}\to\text{UE},n}^{[i,k]}(l-1)| \leq \kappa$ and $|s^{[i,k]'} - s^{[i,k]}(l-1)| \leq \kappa$ **then**
7:       Convergence = **true**
8:       **return** $\{P_{\text{S}\to\text{UE},n}^{[i,k]'}, P_{\text{B}\to\text{S},n}^{[i,k]'}, s^{[i,k]''}\}$
9:    **else**
10:       $P_{\text{B}\to\text{S},n}^{[i,k]}(l) = P_{\text{B}\to\text{S},n}^{[i,k]'}, P_{\text{S}\to\text{UE},n}^{[i,k]}(l) = P_{\text{S}\to\text{UE},n}^{[i,k]'}, s^{[i,k]}(l) = s^{[i,k]''}, l = l+1$
11:    **end if**
12: **until** $l = L_{\max}$

---

that optimization problem (16) is jointly concave with respect to the auxiliary optimization variables and $s^{[i,k]}$. We note that by solving optimization problem (16) for $\widetilde{P}_{\text{B}\to\text{S}}^{[i,k]}, \widetilde{P}_{\text{S}\to\text{UE},n}^{[i,k]}$, and $s^{[i,k]}$, we can recover the solution for $P_{\text{B}\to\text{S}}^{[i,k]}$ and $P_{\text{S}\to\text{UE},n}^{[i,k]}$. In other words, the solution of (16) is asymptotically optimal with respect to (11) for high SNR and a sufficiently large number of subcarriers.

In the following, we propose an algorithm for solving the transformed problem in (16).

D. Iterative Resource Allocation Algorithm

The proposed iterative resource allocation algorithm is based on alternating optimization. The algorithm is summarized in Table I. The algorithm is implemented by a repeated loop. In line 2, we first set the iteration index l to zero and initialize the resource allocation policy. Variables $P_{\text{B}\to\text{S},n}^{[i,k]}(l), P_{\text{S}\to\text{UE},n}^{[i,k]}(l)$ and $s^{[i,k]}(l)$ denote the resource allocation policy in the l-th iteration. Then, in each iteration, we solve (16) for $P_{\text{B}\to\text{S},n}^{[i,k]}$ using (17) with $s^{[i,k]}, \forall i, k$, and $P_{\text{S}\to\text{UE},n}^{[i,k]}(l-1)$ from the last iteration. Then, we obtain an intermediate power allocation variable $P_{\text{B}\to\text{S},n}^{[i,k]'}$ which is used

as an input for solving (16) for $P_{\text{S}\to\text{UE},n}^{[i,k]}$ via (18), c.f., line 5. We note that (17) and (18) are obtained by standard convex optimization techniques while variables λ and β in (17) and (18) are the Lagrange multipliers with respect to constraints C1 and C2 in (16), respectively. The optimal values of λ and β in each iteration can be easily found with a standard gradient algorithm such that constraints C1 and C2 in (16) are satisfied. After we obtain the intermediate solution for power allocation, we use it to derive the optimal allocation of subcarrier i at the BS to UE k which is given by

$$s^{[i,k]*} = \begin{cases} 1 & \text{if } k = \arg \max_{t \in \{1,\dots K\}} \Psi_t, \\ 0 & \text{otherwise}, \end{cases} \quad (19)$$

and

$$\Psi_k = w^{[k]}\left(\sum_{n=1}^{N} \log_2\left(1 + \widetilde{\text{SINR}}_n^{[i,k]*}\right) - \frac{\widetilde{\text{SINR}}_n^{[i,k]*}}{1 + \widetilde{\text{SINR}}_n^{[i,k]*}} \right). (20)$$

$\widetilde{\text{SINR}}_n^{[i,k]*}$ is obtained by substituting the intermediate solution of $P_{\text{B}\to\text{S},n}^{[i,k]'}$ and $P_{\text{S}\to\text{UE},n}^{[i,k]'}$, i.e., (17) and (18), into (16) in the l-th iteration. Then, the procedure is repeated iteratively until we reach the maximum number of iterations or convergence is achieved. We note that the convergence to the optimal solution of (16) is guaranteed for a large number of iterations since (16) is concave with respect to the optimization variables [16]. Besides, the proposed algorithm has a polynomial time computational complexity.

IV. RESULTS AND DISCUSSIONS

In this section, we evaluate the system performance based on simulations. We assume that there are K UEs located in an indoor environment and the BS is located outdoor. For the BS-to-SUDAS links, we adopt the Urban macro outdoor-to-indoor scenario of the Wireless World Initiative New Radio (WINNER+) channel model [17]. The center frequency and the bandwidth of the licensed band are 800 MHz and 10 MHz, respectively. There are 600 subcarriers which are grouped into 50 resource blocks with 12 subcarriers per resource block for data transmission. Each subcarrier has a bandwidth of 15 kHz. Hence, the BS-to-SUDAS link configuration is in accordance with the Long Term Evolutions (LTE) standard [18]. As for the SUDAS-to-UE links, we adopt the IEEE 802.11ad channel model [19] in the range of 60 GHz. There are M subbands. The first subband has a frequency range from 60 GHz to 60.01 GHz and there is a 30 MHz guard band between any two consecutive subbands. The maximum
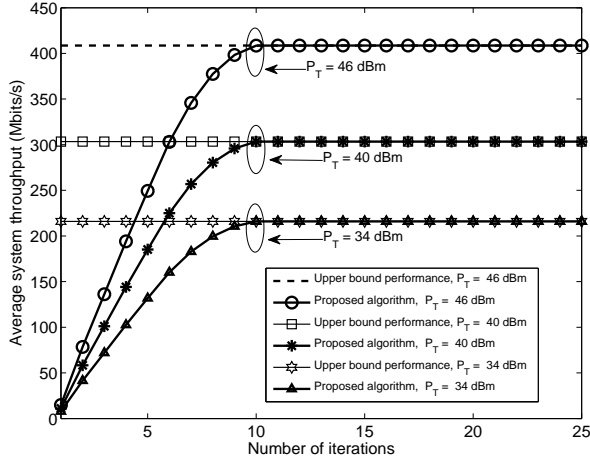
Fig. 3. Average system throughput (Mbits/s) versus the number of iterations for $N_T = 8$ transmit antennas at the BS, $K = 2$ UEs, $M = 8$ SUDACs, and different maximum transmit powers at the BS, P_T.
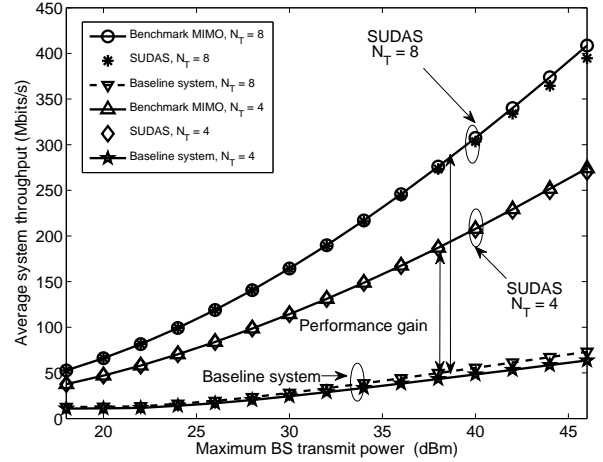

Fig. 4. Average system throughput (Mbits/s) versus the maximum transmit power at the BS (dBm) for $K = 2$ UEs, $M = 8$ SUDACs, and different communication systems. The double-sided arrows indicate the throughput gains achieved by the SUDAS compared to the baseline system.

transmit power per SUDAC is set to $P_{max} = 23$ dBm which is in accordance with the maximum power spectral density suggested by the Harmonized European Standard [20], i.e., 13 dBm-per-MHz. For simplicity, we assume that $w^{[k]} = 1, \forall k$, and $N_S = \min\{N_T, M\}$ for studying the system performance.

A. Convergence of the Proposed Iterative Algorithm

Figure 3 illustrates the convergence of the proposed algorithm for $N_T = 8$ transmit antennas at the BS, $K = 2$ UEs, $M = 8$ SUDACs, and different maximum transmit powers at the BS, P_T. We compare the system performance of the proposed algorithm with a performance upper bound which is obtained by computing the optimal objective value in (16), i.e., assuming noise free reception at the UEs. The performance gap between the two curves constitutes an upper bound on the performance loss due to the high SINR approximation adopted in (15). It can be seen that the proposed algorithm approaches 99% of the upper bound value after 20 iterations which confirms the practicality of the proposed iterative algorithm.

B. Average System Throughput versus Transmit Power

Figure 4 illustrates the average system throughput versus the maximum transmit power at the BS for $K = 2$ UEs, $M = 8$, and different numbers of transmit antennas N_T at the BS. The performance of the proposed algorithm for the SUDAS with 10 iterations is compared with that of a benchmark MIMO system and a baseline system. For the benchmark MIMO system, we assume that each UE is equipped with M receive antennas without the help of the SUDAS and optimal resource allocation is performed to maximize the system throughput. In other words, the average system throughput of the benchmark system serves as a performance upper bound for the proposed SUDAS. As for the baseline system, the BS performs optimal resource allocation and utilizes only the licensed frequency band without the help of the SUDAS and the UEs have only one antenna. As can be observed, the proposed SUDAS is able to exploit the spatial multiplexing gain even though each UE is
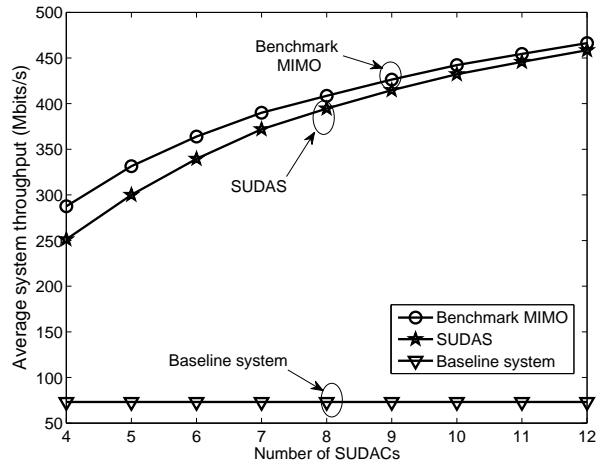

Fig. 5. Average system throughput (Mbits/s) versus the number of SUDACs for a maximum transmit power at the BS of $P_T = 46$ dBm for different communication systems.

equipped with a single antenna. Besides, a huge performance gain is achieved by the SUDAS compared to the baseline system as the SUDAS utilizes both licensed and unlicensed bands. On the other hand, the performance of the proposed scheme and the benchmark system improves rapidly with increasing number of transmit antennas due to more degrees of freedom for resource allocation.

C. Average System Throughput versus Number of SUDACs

Figure 5 depicts the average system throughput versus the number of SUDACs for $N_T = 8$. The maximum BS transmit power is 46 dBm. It can be observed that the system throughput grows with the number of SUDACs. In particular, a higher spatial multiplexing gain can be achieved when we increase the number of SUDACs M if $N_T \geq M$. For $M > N_T$, increasing the number of SUDACs in the system

leads to more spatial diversity which also improves the system throughput.

V. Conclusion

In this paper, we studied the resource allocation algorithm design for SUDAS assisted downlink multicarrier transmission. In particular, the SUDAS utilizes both licensed and unlicensed frequency bands for improving the system throughput. The resource allocation algorithm design was formulated as a non-convex matrix optimization problem. In order to obtain a tractable solution, we revealed the structures of the optimal precoding matrices such that the problem could be transformed into a scalar optimization problem. Based on this result, we proposed an efficient iterative resource allocation algorithm to solve the problem by alternating optimization. Our simulation results show that the proposed SUDAS assisted transmission provides a substantial throughput gain compared to conventional systems which do not utilize the unlicensed frequency spectrum. It is expected that the proposed SUDAS is able to bridge the gap between the current technology and the high data rate requirement of the next generation communication systems.

Appendix-Proof of Theorem 1

Due to the page limitation, we provide only a sketch of the proof which follows a similar proof in [21]. We first show that the optimal $\mathbf{P}^{[i,k]}$ and $\mathbf{F}^{[i,k]}$ jointly diagonalize the end-to-end channels on each subcarrier for the maximization of the objective function in (11). Then, we construct the optimal precoding and post-processing matrices based on the optimal structure. The MSE matrix for data transmission on subcarrier i for UE k can be written as:

$$\mathbf{E}^{[i,k]} = \left(\mathbf{I}_{N_S} + (\mathbf{\Gamma}^{[i,k]})^H (\mathbf{\Theta}^{[i]})^{-1} \mathbf{\Gamma}^{[i,k]}\right)^{-1} \quad (21)$$

$$= \mathbf{I}_{N_S} - (\mathbf{\Gamma}^{[i,k]})^H \left(\mathbf{\Gamma}^{[i,k]}(\mathbf{\Gamma}^{[i,k]})^H + \mathbf{\Theta}^{[i]}\right)^{-1} \mathbf{\Gamma}^{[i,k]}.$$

Since the objective function for each subcarrier is a Schur-concave function, by applying the majorization theory [22], it can be shown that the sum of the diagonal elements of the MSE matrix is minimized when matrix $(\mathbf{\Gamma}^{[i,k]})^H \left(\mathbf{\Gamma}^{[i,k]}(\mathbf{\Gamma}^{[i,k]})^H + \mathbf{\Theta}^{[i]}\right)^{-1} \mathbf{\Gamma}^{[i,k]}$ is a diagonal matrix. In other words, the objective function is maximized when the MSE matrix is a diagonal matrix.

On the other hand, we focus on the power consumption constraints C1 and C2 in (11). For a given subcarrier allocation and a given achievable data rate, it can be shown that the transmit powers at the BS and the SUDAS are minimized when matrices $\mathbf{P}^{[i,k]}$ and $\mathbf{F}^{[i,k]}$ are given by

$$\mathbf{P}^{[i,k]} = \widetilde{\mathbf{V}}^{[i]}_{\text{B}\to\text{S}} \mathbf{\Lambda}^{[i,k]}_{\text{B},k} \quad \text{and} \quad (22)$$

$$\mathbf{F}^{[i,k]} = \widetilde{\mathbf{V}}^{[i]}_{R_m,k} \mathbf{\Lambda}^{[i,k]}_{\text{F},k} (\widetilde{\mathbf{U}}^{[i]}_{\text{B}\to\text{S}})^H, \quad (23)$$

respectively, where all involved matrices are defined after (13). Since both $\mathbf{P}^{[i,k]}$ and $\mathbf{F}^{[i,k]}$ in (22) and (23) jointly diagonalize the end-to-end equivalent channel and achieve the minimum transmit power for any achievable system data rate, (22) and (23) are the optimal precoding and post-processing matrices. ∎

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